QUESTIONS AND ANSWERS
Part II

I list here below my main contributions to Quora, English version, from August 23rd, 2018, to September 4th, 2018. I wrote so far 38 contributions.
I have edited a few answers. Others, the longest ones, I have put (or I plan to put) on separate pages in the present site.

30 agosto 2018

What is your favorite derivation of Lorentz transformation?

I just repeated the answer (which I gave on the same day) to the question:

How are Lorentz transformations derived? Are they just based on the fact that the speed of light is a constant to every observer?

I believe that one thing is the theory of special relativity, and another is the algebraic derivation of the Lorentz transformations, which, indeed, **can be derived based on the sole fact that the speed of light is constant for all observers.**

This is because the Lorentz transformations are not the whole of relativity, and Lorentz himself did not get a grasp of the fundamental physical meaning of the transformations he had found.

Let's consider the rotation of a bar with a fixed end at the origin in the ordinary Euclidean space. If we rotate the bar (with coordinates of the extremes (0, 0) and (x, y)) by a ϑ angle, we have the new coordinates

$$(i) \qquad X = x\,cos\vartheta - \; y\,sin\vartheta$$

$$(ii) \qquad Y = x\,sin\vartheta + y\,cos\vartheta$$

One can easily prove that the length of the bar, $x^2 + y^2 = X^2 + Y^2$, that is, it is invariant.

We can also express the two equations of transformation

(ì and ii) in terms of Tan(ϑ) = y / x, which we will call m.

Using the relations of the elementary trigonometry,

From

$$1 + tg^2\vartheta = \frac{cos^2\vartheta + sin^2\vartheta}{cos^2\vartheta} = \frac{1}{cos^2\vartheta}$$

we obtain $cos\vartheta = \frac{1}{\sqrt{1+m^2}}$, e $sin^2\vartheta = 1 - \frac{1}{1+m^2}$, from which $sin\vartheta = \frac{m}{\sqrt{1+m^2}}$.

The result of the substitutions in (i) and (ii) is:

$$X = \frac{1}{\sqrt{1 + m^2}}(x - m\,y)$$

$$Y = \frac{1}{\sqrt{1 + m^2}}(y + m\,x)$$

I think that at this point those who know special relativity begin to see some similarity to Lorentz transformations.

If we want to indicate that the speed of light remains constant by changing observers, we must have:

$$x'^2 - (ct')^2 = x^2 - (ct)^2.$$

However, observed the mathematician Minkowsky, this relation becomes equal to that for ordinary rotations, if we introduce the variables T = ict, T '= ict'.

That is: (1) $x'^2 + T'^2 = x^2 + T^2$

We have thus entered the Minkowsky space, a pseudo-Euclidean space, which differs from the Euclidean space because we have an imaginary time coordinate, and the distance is no longer positive definite (that is, always positive or zero).

In fact, in Minkowsky's space there is no talk of "distance" but rather of "interval," which can be positive, zero, negative.

The interval expressed by (1) will obviously remain invariant for transformations formally identical to those of the rotations, but with a coordinate, T, imaginary. One can speak of "rotations in space-time."

$$x' = \frac{1}{\sqrt{1 + m^2}}(x - mT)$$

$$T' = \frac{1}{\sqrt{1 + m^2}}(T + m\,x)$$

Having introduced imaginary values, we must now take them into account. For the rotations, we had taken m= y / x, but if we had chosen x / y, our formulas would not have been affected that much: we would have only had to exchange the order of the two coordinates. So we choose m = x / T = x / ict = -iv / c = -i β

Next we replace imaginary values everywhere. The first equation becomes:

(i ') $$x' = \frac{1}{\sqrt{1-\beta^2}}\left(x - (-\,\mathrm{i}\,\beta)(\mathrm{ict})\right) = \frac{1}{\sqrt{1-\beta^2}}(x - vt)$$

Secondly, from

$$ict' = \frac{1}{\sqrt{1-\beta^2}}(ict \;\; - \mathrm{i}\,\beta x)$$

dividing both members by ic:

(ii') $$\frac{1}{\sqrt{1-\beta^2}}\left(t \;\; - \frac{v}{c^2}\,x\right)$$

And with this we have the Lorentz transformations. The method is formally correct and allows to calculate a correct relativistic sum of the velocities. In addition, it can be developed in terms of hyperbolic functions, and allows to define something similar to the ϑ angle, which had the advantage of being additive while its trigonometric tangent was not, and so forth.

Nowadays, Minkowsky's space is no longer in fashion, especially since it cannot be extended (as far as I know) to general relativity. But if one does not intend to study general relativity and does not care to learn the necessary rudiments of tensor calculus, Minkowsky's space offers, I believe, the most straightforward formal derivation of the Lorentz transformations, a **transformation that is based solely on the constancy of the velocity of the light** and in my opinion is "unforgettable".

(The first time I saw this approach was in "Spacetime Physics," by E.F Taylor and J. A. Wheeler, 1st ed. 1968. Wheeler himself waved “goodbye to ict” in the classic “Gravitation,” by C.W.Misner, K.S.Thorne, J.A.Wheeler (1975) p.51)

Updated August 30, 2018

Why does $\lim_{h\to 0}\frac{Cosh\,(h)-1}{h}$ while $\lim_{h\to 0}\frac{Sinh\,(h)}{h}$ =1

In this type of questions, referring to *elementary transcendental functions*, I always use their series expansion.

cosh = $1+h^2/2$ + higher order terms (which go faster than h^2 to zero), therefore cosh-1 =$+h^2/2$, which, divided by h, gives +h/2, which goes to zero as h goes to zero.

On the other hand, Sinh = $h+h^3/6$…, which divided by h goes to 1 - terms which go to zero faster than h.

August 25, 2018

Why, in the end, did Otto von Bismarck resign?

Besides doing some editing, I attach to my answer the resignation letters of 18 March 1890.

Bismarck was used to ruling. On his headstone, it is written, according to his wishes: "Faithful servant of Emperor William I," not William II. In reality, Bismarck took advantage of an ordinance of the Kingdom of Prussia (September 8, 1852), that the members of the Council of Ministers could not speak to the King/Emperor except through the Minister President / Chancellor. William I, grateful to Bismarck for becoming Emperor in 1871 (he was already 74 at that time), had some arguments with him, but invariably supported him. It can be said that actually Bismarck was the ruler for twenty-eight years, first in the Kingdom of Prussia (1862-1873, official date) and then in the German Empire (1873-1890).

Very often he had to fight against an adverse parliament, both in Prussia and in the Empire, a situation which did not impress him in the least, as he considered himself solely responsible to William I.

I think that his personal politics had three fundamental points: (1) the ambition to be in command, (2) the greatness (not the overbearing power) and the security of Germany (the second entailed the need to isolate France); (3) fidelity to the Emperor (at least until he obeyed him).

But ...

William I died in 1888, and his son Friedrich III, who did not like Bismarck, was his successor. But Friedrich III died after three months of reign, and his son William II came to power. Bismarck, it is argued, was not prepared to interact with William II. It seems unlikely to me, because Frederick III was already gravely ill when he took power. Instead, I think it more likely that William II had already decided for some time to get rid of Bismarck at all cost.

The new emperor, twenty-nine, was obviously not accustomed to be in command, but, ambitious and inexperienced as he was, he was determined to rule, which meant, as Bismarck had predicted, that he would fall into the hands of incompetent courtiers (political advisors, and worse still, military).

Furthermore, William II felt no cause for gratitude for Bismarck. *At this point, I think it was just a matter of pretexts and time, to force Bismarck to resign, the first task that the Emperor had clearly set to himself.*

Virtually the Emperor showed his disagreement with Bismarck on every point that Bismarck considered fundamental to German politics. There were big and small matters, of which I will only enunciate the principal ones, also because I think that the struggle for the real power, in which Bismarck could only lose, was the real reason that was at the base of all the disagreements between the two, that had to lead to Bismarck's dismissal-resignation inevitably.

The points of greatest disagreement were:

1. **The imperialism** (*Weltpolitik*) to which the young emperor tended, opposing Bismarck's cautious policy of equilibrium.

2. **The secret and therefore risky "*Treaty of Reinsurance*" (1887)**, wanted by Bismarck, which allowed Germany not to be against Russia. Instead William II wanted absolute loyalty to the Austrian Empire, for example in case of contrasting views or even an Austro-Russian war.

3. **Attitude towards socialism**. In 1890 Bismarck attempted to pass a strongly anti-socialist law, with the probable aim of provoking a socialist uprising, which would have been suppressed by force. William, more moderate than Bismarck on this issue, refused, because "he did not want to start his reign with a bloodbath of his subjects". In the frantic negotiations about the anti-socialist law, Bismarck's ruling party, the so-called *Kartell*, did not succeed to support him, and his government fell.

4. The last straw was Bismarck's attempt to regain power by **forming an anti-socialist coalition with the center (Catholics) and conservatives**. When he learned of a meeting between Bismarck and Windthorst, the head of the Catholic party, the emperor was furious. On March 15, 1890, in the Radziwill Palace, Bismarck's residence, a stormy meeting (according to witnesses who were not present) took place, the outcome of which was foregone: the emperor had not yet learned to moderate his feelings, and Bismarck, seventy-five, could no longer moderate his own. Before going out in anger, the Emperor ordered the abolition of the above-mentioned ordinance of 1852, which prevented ministers from reporting directly to the Emperor. After the meeting, William II insistently asked for the resignation of the Chancellor, who at last yielded. The letter of resignation signed by Bismarck bears the date of 18 March 1890, a bitter letter (see Appendix), which

was published only after his death. On March 20, he officially resigned and received the imperial order to withdraw to his property. The intellectual part of the public responded with relief at his resignation. The commoners seem to have looked at him with greater sympathy: when Bismarck left his residence in Berlin, a large crowd accompanied his carriage to the station, where an even greater crowd was waiting, and, although such a gathering was forbidden, the police he did not intervene.

As a sop, he was promoted to "Colonel General with the dignity of Marshal" and was named Duke of Lauenburg. He commented that such a title would be useful to him when he wanted to travel incognito.

After an attempt to be elected to the Reichstag in 1891 (he was, but only after a ballot that humiliated him, so he never participated in the sessions of Parliament), in 1892 he made a trip to Vienna for the marriage of his son Herbert. During the journey he was hailed by the crowd, also in Dresden and Vienna (where he did not expect to be so popular). There are various conflicting reports about his stay in Vienna. It seems that the Court ignored him. He declared that with the German government "All the bridges had been cut".

On 27 January 1894 in Berlin there was a sort of reconciliation between William II and Bismarck, who had been invited for the Emperor's birthday. Wilhelm received him informally at the station, and kissed him on both cheeks. But it seems that this "reconciliation" was nothing more than a public relations show, which did not have, nor could have any political consequences.

He had retired to Varzin in Pomerania, but, a month after his wife's death (November 27, 1894) he moved to Friedrichsruh near Hamburg, where he lived in retirement writing his memoirs, and always waiting to be called to power in some form. The Emperor actually visited him a few times, but the expected call never came, in any form.

Bismarck died on the evening of July 30, 1898.

He had predicted that if Germany continued to proceed on the path taken after his dismissal, disaster would strike twenty years after his death. The facts gave him reason: on November 11, 1918, Germany surrendered by putting an end to the disastrous First World War.

NOTE. At this point, it seems appropriate to insert an unusual portrait of Bismarck, which makes me meditate. File:Bismarck11Jahre.jpg - Wikimedia Commons

File:Bismarck11Jahre.jpg, Public Domain
Otto von Bismarck at 11 (chalk drawing by Franz Krueger, Berlin 1826)

APPENDIX.

I attach here an English version of the famous letter from Bismarck dated March 18, 1890. The source is http://germanhistorydocs.ghi-dc.org/docpage.cfm?docpage_id=2906

Wikipedia , English edition, normally reliable, defines it as "blistering letter". I would rather define it "bitter".

It highlights the importance of the famous 1852 ordinance, which allowed Bismarck to rule with the sole support of the King, then Emperor, William I. In the words of Bismarck, "his very existence [of the ordinance] and awareness to have the trust of their Majesties, Wilhlem and Friedrich, was enough to assure my authority over my staff. " Moreover, complained about the separation of the offices of Minister of Prussia and Chancellor of the Reich, Bismarck protests for the new imperial directives of foreign policy especially with Russia, over which it had not been consulted. Then he asks for leave.

Berlin, 18 March 1890.

At my respectful audience on the 15th of this month, Your Majesty commanded me to draw up a decree annulling the All-Highest Order of September 8, 1852, which regulated the position of the Minister-President vis-à-vis colleagues.

May I, your humble and most obedient servant, make the following statement on the genesis and importance of this order:

There was no need at that time of absolute monarchy for the position of a "President of the State Ministry." For the first time, in the United *Landtag* of 1847, the efforts of the liberal delegate (Mevissen) led to the designation, based on the constitutional needs of that day, of a "Premier-President," whose task it would be to supervise uniform policies of the responsible ministers and to take over responsibility for the combined political actions of the cabinet. In the year 1848, this constitutional practice was introduced into our system, and "Presidents of the State Ministry" were appointed, such as Count Arnim, Camphausen, Count Brandenburg, Baron von Manteuffel, and Prince von Hohenzollern, who were primarily responsible not for one portfolio, but rather for the overall policy of the cabinet and thus for all the portfolios. Most of these gentlemen did not hold portfolios of their own, but rather only the presidency, as was most recently the case, before my assumption of the post, with Prince von Hohenzollern, the Minister von Auerswald, and Prince Hohenlohe. It was incumbent on them, however, to ensure that the State Ministry maintained – both within itself and in its relationship with the monarch – the kind of unity and steadiness that is absolutely required of any ministerial responsibility that forms the basis of constitutional life. The relationship of the State Ministry and its individual members to the new institution of the Minister-President very quickly required a new constitutional regulation, which was effected with approval of the then State Ministry by the order of September 8, 1852. Since then, this order has been decisive in regulating the relationship of the Minister-President and the State Ministry, and it alone gave the Minister-President the authority which enabled him to take over responsibility for the policies of the cabinet, a responsibility demanded by the *Landtag* as well as public opinion. If each individual minister must receive instructions from the monarch, without previous understandings with his colleagues, it becomes impossible in the cabinet to sustain uniform policies, for which each member can be responsible. There remains for none of the ministers and, especially, for the Minister-President any possibility of bearing constitutional responsibility for the whole policy of the cabinet. [In the absolute monarchy, a regulation such as contained in the order of 1852 is dispensable and would be so today if we returned to absolutism without ministerial responsibility; according to the rightly existing constitutional institutions, however, a presidential leadership of the ministerial committee based on the principle of the order of 1852 is indispensable. On this point, all of my colleagues agree, as was ascertained at yesterday's meeting of State Ministers; they also agree that any successor of mine in the ministerial presidency would not be able to bear the responsibility for his office if he lacked the authority that the order of 1852 confers. For any of my successors, this need will be even more pronounced, because he will not immediately be supported by the authority that my presidency of many years and the trust of the two late emperors have granted to me.] Up to this time, I have never felt the need, in my relationships with my colleagues, to draw upon the order of 1852. Its very existence and the knowledge that I possessed the confidence of their late Majesties, Wilhlem and Friedrich, were enough to assure my authority on my staff. This knowledge exists today neither for my colleagues nor for myself. I have been compelled, therefore, to turn back to the order of 1852, in order to assure the necessary uniformity in the service of Your Majesty. On the aforementioned grounds, I am not in a position to carry out Your Majesty's demand, which would require me to initiate and countersign the suspension of the order of 1852 recently brought up by me, and, despite that, at the same time carry on the presidency of the Ministry of State.

According to the information conveyed to me yesterday by Lieutenant General von Hahnke and Cabinet Privy Councilor von Lucanus, I can have no doubts that Your Majesty knows, and believes, that it is impossible for me to rescind the order while at the same time staying on as Minister-President. Nevertheless, Your Majesty has upheld the command given to me on the 15th of March and has held out the prospect of granting my request for dismissal.

After past discussions with Your Majesty on the question of whether my remaining in office would be unwelcome to Your All-Highest Majesty, I had reason to assume that Your All-Highest Majesty would be pleased if I gave up my positions in His Prussian services but continued on in Reich services. Upon closer examination of this question, I took the liberty of drawing attention, with all due reverence, to a number of serious consequences that would result from the separation of my offices, especially with regard to the future appearance of the Chancellor in the Reichstag, and I will refrain from repeating all of the consequences that such a separation between Prussia and the Reich Chancellor would have. As a result, Your Majesty deigned to grant permission to "leave things as they are" for the time being. However, as I had the honor of explaining, it is not possible for me to maintain the office of a Minister-President after Your Majesty has repeatedly ordered the *capitis diminutio* (reduction of authority) entailed by the annulment of the fundamental order of 1852.

During my reverent report on the 15th of March, Your Majesty also deigned to place restrictions on the expansion of my official privileges, thereby leaving me without the degree of participation in state affairs and the oversight of the latter, and without the degree of freedom in my ministerial decisions and in my dealings with the Reichstag and its members, that I require to assume constitutional responsibility for my official activity.

However, even if it were feasible to make our foreign policy as independent from our domestic policy and our Reich policy as independent from our Prussian policy as would be the case if the Reich Chancellor were just as uninvolved in Prussian politics as he is in Bavarian or Saxon politics, and if he had no share in the arrangement of the Prussian vote vis-à-vis the Federal Council and the Reichstag, it would still be impossible for me to implement the orders stipulated by Your Majesty with regard to foreign policy. It would be impossible after Your Majesty's recent decisions on the direction of our foreign policy, as summarized in the imperial billet which Your Majesty enclosed with the reports that were returned to the consul in Kiev yesterday. If I were to do so, I would call into question all of the important successes attained for the German Reich under a foreign policy in keeping with the wishes of Your Majesty's two late successors, all of the successes attained over decades, and under unfavorable conditions, in our relations with Russia, and whose great significance, beyond all expectations, for the present and the future, was just confirmed to me by Count Shuvalov after his return from St. Petersburg.

Considering my attachment to service for the monarchy and for Your Majesty and the long-established relationship which I had believed would exist forever, it is very painful for me to terminate my accustomed relationship to the All Highest and to the political life of the Reich and Prussia; but, after conscientious consideration of the All Highest's intentions, to whose implementation I must always be ready to act, if I am to remain in service, I cannot do other than most humbly request Your Majesty *to grant me an honorable discharge with legal pension from the posts of Reich Chancellor, Minister-President, and Prussian Minister for Foreign Affairs.*

Judging from my impressions over the past weeks and from the revelations of which I learned yesterday from communications by Your Majesty's Civilian and Military Cabinet, I have reason to reverently assume that I am accommodating the wishes of Your Majesty with my request for discharge, and thus I am able to rely with certainty on the gracious approval of my request. I would have submitted the request for dismissal from my offices to Your Majesty earlier, had I not been under the impression that it was Your Majesty's wish to make use of the experience and talents of a loyal servant to His ancestors. Now that I am certain that Your Majesty does not require these, I may withdraw from political life without fearing that my decision will be condemned as untimely by public opinion.

Source of English translation: A portion of this translation was taken from Louis L. Snyder, ed., *Documents of German History*. New Brunswick, NJ: Rutgers University Press, 1958, pp. 266-268. Passages omitted from Snyder's anthology were translated by Erwin Fink for German History in Documents and Images and added to Snyder's translation.

Original German text printed in Otto von Bismarck, *Die gesammelten Werke* [*Collected Works*], ed., Gerhard Ritter and Rudolf Stadelmann, Friedrichsruh ed., 15 vols., vol. 6c, no. 440, Berlin, 1924-1935, pp. 435ff. Original German text republished in Otto von Bismarck, *Werke in Auswahl* [*Selected Works*], ed. Gustav Adolf

Rein et al., 8 vols., vol. 7, *Reichsgestaltung und Europäische Friedenswahrung* [*Formation of the Reich and Keeping Peace in Europe*], pt. 3, 1883-1890, ed. Alfred Milatz. Darmstadt: Wissenschaftliche Buchgesellschaft, 2001, pp. 758-61.

August 25, 2018

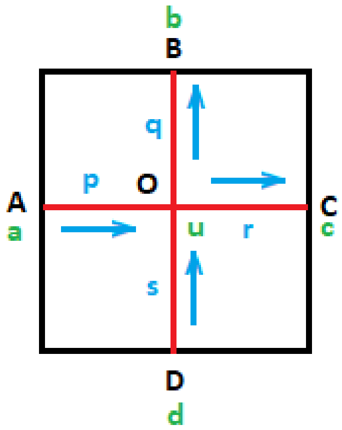
What is an intuitive explanation of the Laplace operator (or Laplacian operator)?

The definitions and mathematical proofs given in the other answers are indisputable. I doubt, however, that they answer the question if the applicant wants to have an intuitive idea of the Laplacian, traditionally represented with Delta (f (x, y, z)).

Of course, I don't know from which level of knowledge of mathematics I have to start, and therefore I assume a high-school level. I will use an example that I found in the work of my favorite math divulger, W.W. Sawyer (A Path to Modern Mathematics, 1966).

Let us suppose we choose an example in two dimensions: a copper sheet with batteries connected to points on its contour. As a result, an electric current will flow through the sheet. We want to investigate the two-dimensional spatial distribution of this current. For an elementary treatment, suppose we replace the sheet with a very fine square-mesh copper net. As a handkerchief resembles (from afar) a continuous sheet, so our copper net will resemble a sheet of copper, provided the meshes are small enough. It is reasonable to think that the distribution of currents in the network could in this case closely approximate the distribution of currents in the sheet. But, whether they are minute or not, we can think of enlarging the meshes to draw the net on a squared sheet on which we will mark an origin and two axes, x and y, parallel to the perpendicular sides of the meshes.

At this point, to better follow the reasoning, it would be good to take paper and pencil and draw a cross, with center on the origin O, and the four points at distance h from O on the four arms of the cross, precisely A to the west, B to the north, C to the east, D to the south. It's all you need.



So let's see how the electric current flows in the network. Two straightforward laws apply:

The first law can be called the "continuity equation." It states that electrical currents flow in the network like water through a network of pipes. In particular, choosing a point on the network as O origin, with 0,0 coordinates, four currents will come or go from point O.

For example:

i) entering from point A (-h, 0), current p;
ii) going out towards point B (0, h), current q;
iii) going out towards point C (h, 0), current r
iv) entering from point D (0, -h), current s.

Here, h is the length one side of the square mesh. *Now, if there are no sources nor sinks of water (or electricity) in O, the sum of the incoming currents must be equal to that of the outgoing currents. So, in our case,*

$$p + s = q + r,$$

which is our rudimentary form of the "*continuity equation*".

The second law is Ohm's Law. At each point (x, y) we associate a potential V (x, y) (for points we mean only the points of the network that have whole coordinates, at the intersections of the threads of the mesh). The potential is like the elevation to water: water flows from the points that have greater elevation to those that are lower; likewise, the electric current flows from the points that have the higher potential to those that have the lower potential. We call u, a, b, c, d the potential in points O, A, B, C, D.

If a current p enters O from A, this means that a > u. The potential drop between A and O is a-u volt. Ohm's law states that the difference in potential between two points A and O (volts) is proportional to the current I (Ampere):

$$\Delta V = R\ I \text{ (where R is the resistance and I the current)}$$

For simplicity, suppose R = 1 unit of resistance, and therefore the current p is not only proportional to the potential difference but is equal to that difference. This applies to all currents. Hence:

$$p = a-u; \quad q = u-b; \quad r = u-c; \quad s = d-u.$$

Substituting in the equation p+s = q+r, we obtain: (a-u)+(d-u) = (u-b)+(u-c).

And then, combining "appropriately" the terms, we get:

I. ((c-u) - (u-a))+((b-u) - (u-d)) = 0

"Appropriately" here means that we have noticed that a, u, c lie on the x-axis; b, u, d lie on the y-axis. Nothing else has changed.

2. CALCULUS NOT REALLY INFINITESIMAL. At this point, we show an example of infinitesimal calculus not really infinitesimal, for beginners. Suppose we have a function that assumes values a, b, c, d at regular intervals h.

For example, we measure the distance traveled by a sprinter, one of those that run a hundred meters in ten seconds.

Seconds Distance
1 …….. a = 8 (start)
2 .... b = 17
3. .... c = 28
4. .... d = 38
5. .... e = 48
6. .... f = 58
7. .... g = 68
8. .... h = 78
9. .... i = 88
10. .... j = 100 (final shot)

We can calculate the average speed on the 100 meters, and we get 10 m / s. But we can be more precise and calculate the speed by measuring the space traveled in a given time h. For example, we choose h = 5 seconds. Then we see that after 5 seconds the average speed is given by the distance traveled in 5 seconds (e-0) / 5 = 48/5 = 9.7 m / s, while after 10 seconds the speed in the last 5 seconds is given by (j-e) / 5 = 52/5 = 10.5 m / s. We always put at the numerator the distance traveled in a given time interval and the time interval itself at the denominator.

We can think of having more and more precise values of the speed, if we reduce the time interval to 2 seconds, to 1 second. But we see that by choosing increasingly smaller time intervals, the velocity does not go to zero, as probably all the Greeks excepting Archimedes would have expected. The other Greeks would have expected that in intervals of time tending to zero the distance traveled would tend to zero, and the velocity would be zero, so that motion would be impossible (one of their - false - paradoxes).

Instead, if we measure the space traveled every 2 seconds, in the first two seconds the speed is (b-0) / 2 = 17/2 = 8.5 m / s, but after another two seconds it is (d-b) / 2 = 11 m / s. If we measure the space traveled every second, we have (a-0) / 1 = 8, (b-a) / 1 = 9, (c-b) = 11.

This is where Calculus starts: decreasing the travel time also decreases the distance traveled, but the ratio distance/time does not necessarily go down to zero: it goes to zero only if the runner stops a few seconds on the road, which normally does not happen.

In conclusion, the speed over a period of time h is given by (a-0) / h, b-a) / h, (c-a)/h for smaller and smaller h. The limit, which such ratios approach if we decrease the time interval is called the **first derivative** of the space function with respect to the time variable.

We have to take another step if we want to get to the Laplacian. We can also take the **second derivative** of space with respect to time. The method is simple: we must subtract not the distance traveled but the consecutive first derivatives, and divide the difference by the same time interval h. For example, an approximation of the second derivative with respect to time in second 2 will be: ((c-b) / h - (b-a) / h) / h = (c-2b+a) / h^2. Of course, the true second derivative will be obtained by calculating the limit of the ratio, with the numerator corresponding to ever smaller h.

Does such an approximate derivative tell us anything? Evidently, it tells us if the "curve" (actually a broken line) for which we calculate it in b is concave or convex: we see that the numerator c -2b+a can be written as 2 ((c+a) / 2 -b). If the numerator is negative, it means that b is greater than the average of c and a, that is the p value that the broken line would assume if a, p, c were aligned, in which case the broken line has a maximum in b. To convince ourselves that the point in the middle, if the points were aligned, would have value (a+c) / 2, we only have to make a proportion: p-a is to h like c-a is to 2h. That is: (p-a) / h = (c-a) / 2h, from which p-a = c / 2 - a / 2 and finally p = (c+a) / 2.

BACK TO WHERE WE WERE

Equation I, though it was obtained in such a naive way, gives us some precious information.

((c-u) - (u-a)) + ((b-u) - (u-d)) = 0

1) Dividing by h^2 (which we can always do) we have:

(c-2u+a) / h^2 + (d-2u b) / h^2 = 0

But this, if we compare it with our "infinitesimal not infinitesimal calculus", is nothing but the sum of the "approximate second derivatives of potential" along the x-axis and along the y-axis, which, going to the limit as we learn the second year of mathematical analysis, are nothing more than the expression:

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$$

or, even better,

Laplacian of the potential = ΔV= 0 (in two dimensions). The extension to the three-dimensional case is not difficult.

Therefore, if in a given region there are no sources of currents, we will have that **ΔV = 0** (where the uppercase Delta is the traditional symbol of the Laplacian). As a result, frankly, it is not too bad, even if it was obtained in a heuristic and not very rigorous way (but I am convinced that in the back of the mind of those who first found that result there was similar reasoning). The solution of the equation is the function V, with appropriate boundary conditions, and we can reconstruct the current distribution from it.

Besides, equation I tells us something more:

2) Bringing the u of equation I to the second member we have: a+b+c+d = 4u, that is u = (1/4) (a+b+c+d), which means that the potential in O is the average of the potential values of the four contiguous points.

So, we have an indication concerning a function in a region in which it respects the equation **Δ(V) = 0**. Since u is the average of the potentials in the four contiguous points (a, b, c, d), it cannot be a maximum of the function, because, to be such, it should be greater than any of them. This applies to any function f(x,y) for all points in the region in which Δ(f (x, y)) = 0.

For an elementary treatment of the Laplacian, due to which Pierre-Simon, Marquis de Laplace (1749-1827) turned in his grave (but maybe not, actually Pierre-Simon de Laplace smiled) I think it should be enough. I only regret not being the creator of this approach.

August 25th, 2018

If 2^x+2=k, what is the value of x in terms of k?

x = Log(k-2)/Log2 + 2 π i n/Log 2 (n must be integer and Log is the natural logarithm).

Having submitted this answer I was asked to explain the steps leading to it. I answered: *I see that the answer you wish is given by Goh Kim Tee. The imaginary part which I have added comes from the definition of logarithm extended to the complex numbers. I don't know whether you are asked to include it in the answer.*

Goh Kim Tee had answered:

2^x=(k-2)

x Log 2= Log (k-2)

x= Log(k-2)/Log 2

Updated August 29, 2018

Does Newton's first law of motion define force?

The definition of force, actually of three types of force, is given on page 1 of the book **PHILOSOPHIÆ NATURALIS PRINCIPIA MATHEMATICA** (Latin edition 1687), in the section **Definitions**, which follows the (differently numbered) title page, author’s preface, and a poem in latin hexameters by Edmund Halley.

Summary of the section “Definition”

(Definition 1: Quantity of matter; Definition II: Quantity of motion)

Definition III: Vis insita, force of inertia; Definition IV: Vis impressa (impressed force); Definitions V, VI, VII, VIII: Centripetal force (such as gravity). Furthermore Definition VIII elaborates the concepts of the three types of forces (motive, accelerative and absolute).

The ensuing Scholium says more about concepts mentioned in the definitions: absolute time, absolute space, absolute place, absolute motion. The famous experience of the pail to make a distinction between absolute and relative circular motion is also mentioned here.

The three laws of motion are given in the next section: Axiomata (Axioms or laws of motion). Three laws, and six corollaries, all referring to the definitions previously given.

For example, the first law is:

Corpus omne perseverare in statu suo quiescendi vel movendi uniformiter in directum, nisi quatenus a viribus impressis cogitur statum illum mutare.

*Every body perseveres in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by **forces impress’d** thereon.*

Which means that the concept of force is assumed to be known from the definitions, in particular

Def. IV: Vis impressa, impressed force

Vis impressa est actio in corpus exercita, ad mutandum ejus statum vel quiescendi vel movendi uniformiter in directum.

Consistit hæc vis in actione sola, neq; post actionem permanet in corpore. Perseverat enim corpus in statu omni novo per solam vim inertiæ. Est autem vis impressa diversarum originum, ut ex ictu, ex pressione, ex vi centripeta.

An impress’d force is an action exerted upon a body, in order to change its state, either of rest, or of moving uniformly forward in a right line.

This force consists in the action only; and remains no longer in the body, when the action is over. For a body maintains every new state it acquires, by its *Vis Inertiæ* only. Impress'd forces are of different origines; as from percussion, from pressure, from centripetal force.

The reader has the choice to decide whether all definitions are satisfactory to him or not.

I write no opinion of mine, only that they were satisfactory to Newton.

Vale.

August 23rd, 2018

What is the limit as x approaches 0 of (1-cos3x) /sin2x?

As usual, I'm in favor of series developments:

$1 - \cos 3x = 1 - 1 + 9x^2/2$ plus terms of the fourth order in x

$\sin 2x = 2x$ plus terms of the third order in x

The ratio is $9x^2/(4x)= 9x/4$, which goes to zero for $x \to 0$.